Intro. to Waves.

1-D Wave Equation:

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{1}{v^2}\,\frac{\partial^2 \Psi(x,t)}{\partial t}, \quad (1)$$

where v is the speed of wave propagation.

General Solution to (1) is given by

$$\Psi(x,t) = f(x-vt) + g(x+vt), \quad (2)$$

where f and g are <u>any</u> functions

f represents a waveform traveling in the <u>positive</u> x direction.

g represents a waveform traveling in the <u>negative</u> x direction.

Let: $\Psi(x,t) = A\cos(kx-\omega t)$

Note: Your Book chooses $\Psi(x,t) = A\sin(kx-\omega t)$.

then $\frac{\omega}{k} = v$ [why? Hint: $A\cos k(x-\frac{\omega}{k}t)$]

Some Terminology:

$A \Leftrightarrow$ Amplitude ; $k \Leftrightarrow$ Wavenumber ; $\omega \Leftrightarrow$ Angular frequency.

$k = \frac{2\pi}{\lambda}$, $\lambda \Leftrightarrow$ wavelength

$\omega = \frac{2\pi}{T}$, $T \Leftrightarrow$ period

$\nu = \frac{1}{T}$, $\nu \Leftrightarrow$ frequency

Generic Wave: $A\cos(kx-\omega t+\delta)$

$\delta \leftrightarrow$ phase constant

The argument of the cosine is called phase; do NOT confuse it with "phase constant"

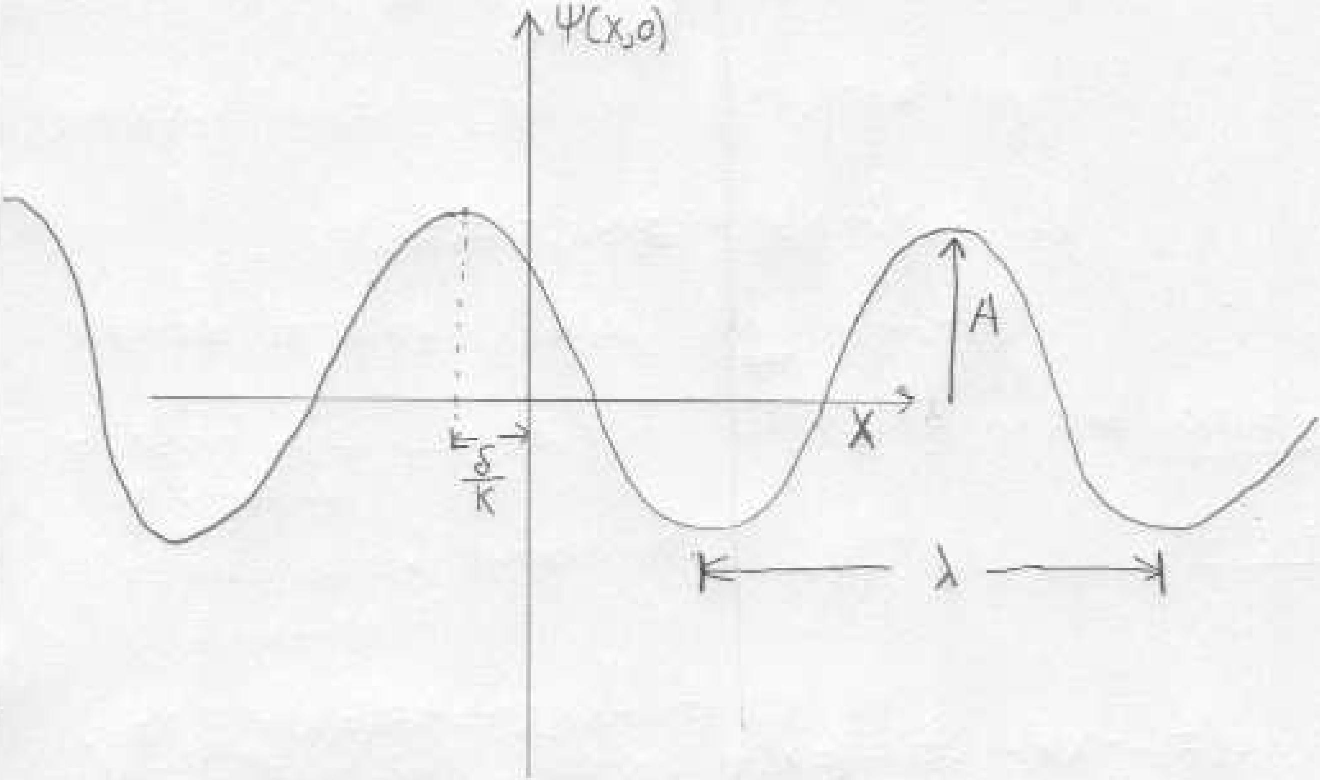
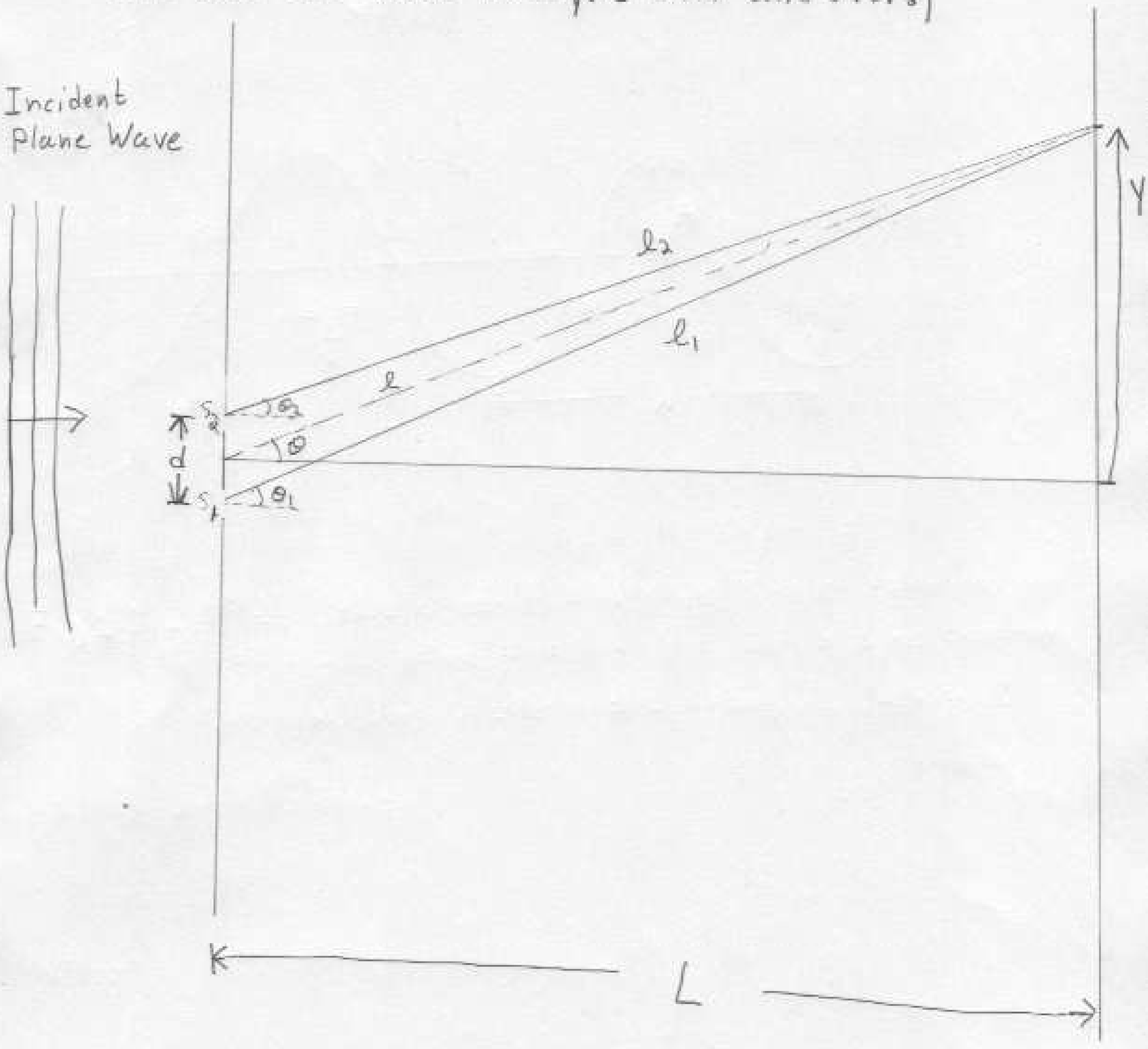


Fig 1: If $\delta=0$, the central maximum passes the origin @ $t=0$. $\frac{\delta}{k}$ is the distance by which the central maximum (and $\therefore$ the entire wave) is "delayed".

Interference: Key Characteristic of Wave Phenomenon

Young's Double-Slit Experiment with Light.

(Done in great detail here. Wake-up because we will use the "same" analysis over and over.)



$d << L$ (3)

Find a condition for a maximum (constructive inteference)?

1. Show ℓ_2 is "about" parallel to ℓ_1 and ℓ, i.e., show they have the "same" slopes.

ℓ:

$$\tan(\theta) = \frac{y}{L} \quad (4)$$

ℓ_1:

$$\tan(\theta_1) = \frac{y + \frac{d}{2}}{L}$$

$$\tan(\theta_1) = \frac{y}{L} + \frac{d}{2L}$$; use (3) to obtain

$$\tan(\theta_1) \approx \frac{y}{L} \quad (5)$$

ℓ_2:

$$\tan(\theta_2) = \frac{y - \frac{d}{2}}{L}$$

$$\tan(\theta_2) = \frac{y}{L} - \frac{d}{2L}$$; use (3) to obtain

$$\tan(\theta_2) \approx \frac{y}{L} \quad (6)$$

Comparing (4), (5) and (6) we conclude that line ℓ, line ℓ_1, line ℓ_2 are "about" parallel, i.e.,

$\theta \cong \theta_1 \cong \theta_2$ (7).

2. What is the extra distance that light from S_1 has to travel?

$$\ell_1 = \sqrt{L^2 + \left(y + \frac{d}{2}\right)^2} \quad (8)$$

$$\ell_2 = \sqrt{L^2 + \left(y - \frac{d}{2}\right)^2} \quad (9)$$

Use: $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \cdots, \ |x| < 1$

$$\ell_1 = L\sqrt{1 + \frac{\left(y+\frac{d}{2}\right)^2}{L^2}} \approx L\left[1 + \frac{\left(y+\frac{d}{2}\right)^2}{2L^2}\right]$$

$$\ell_2 = L\sqrt{1 + \frac{\left(y-\frac{d}{2}\right)^2}{L^2}} \approx L\left[1 + \frac{\left(y-\frac{d}{2}\right)^2}{2L^2}\right]$$

$$\ell_1 - \ell_2 = \Delta\ell = L\left[\frac{yd}{2L^2} + \frac{yd}{2L^2}\right]$$

$$\Delta\ell = \frac{yd}{L} \quad (10)$$

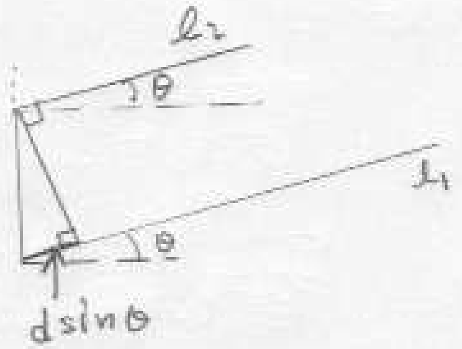
Note: $\tan\theta = \frac{y}{L}$, thus,

$$\Delta\ell = d\tan\theta \quad (11)$$

For small θ, $\tan\theta \approx \sin\theta$ (why?). Thus, (11) can be written as

$$\boxed{\Delta\ell = d\sin(\theta)} \quad (12)$$

Pictorially, $d\sin\theta$ is



For a maximum (constructive inteference) to occur we must fit integer number of wavelengths into the "extra" distance (why?).

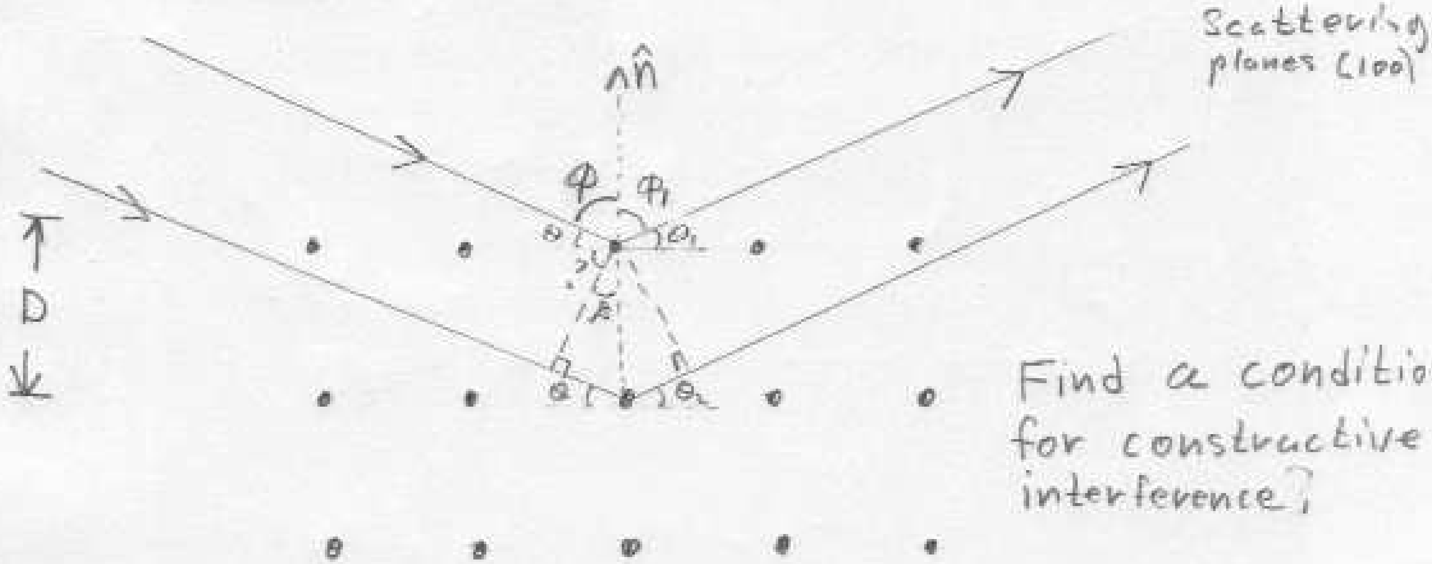
$$\Rightarrow \boxed{d\sin\theta = n\lambda\,, \quad n = 0, 1, 2, 3, \ldots} \qquad (13)$$

X-ray Interference From Crystals.

$\lambda_{x\text{-}ray} \sim \mathring{A}$ (atomic Scale). Note: $1\mathring{A} = 10^{-10}$ m.

Crystals act as Diffraction gratings.

1. The angle of incidence = The angle of reflection.
   Note: Angles are measured from the normal of the scattering plane.



Find a condition for constructive interference?

By 1.: $\varphi_1 = \varphi_1 \Rightarrow \theta = \theta_1$ and $\theta = \theta_2$

$\theta + ? = 90 \Rightarrow ? = 90 - \theta$

$? + \beta = 90 \Rightarrow \beta = 90 - ? = 90 - (90 - \theta)$

$\Rightarrow \beta = \theta$

Exra distance:

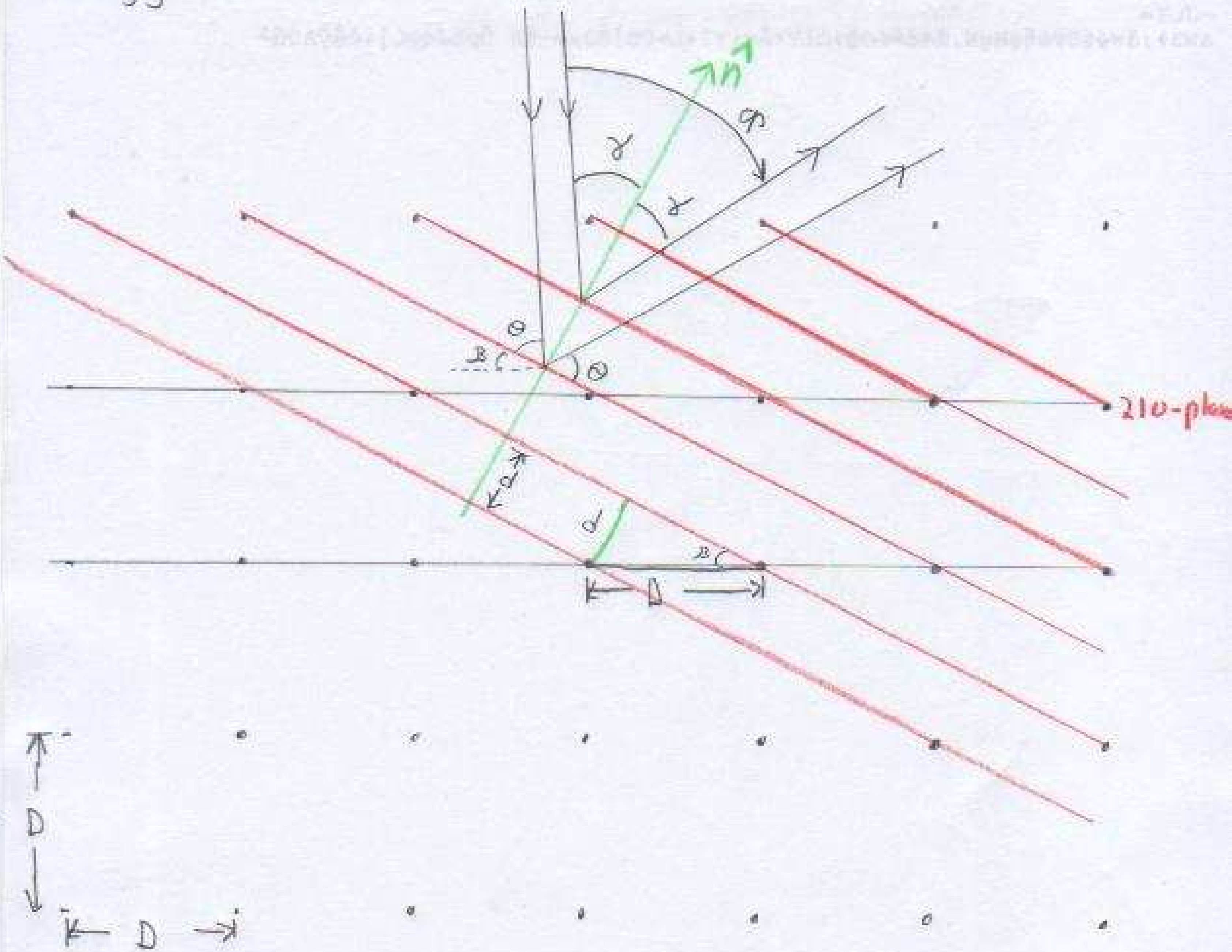
$2D\sin\theta$

Thus,

$\boxed{2D\sin\theta = n\lambda}$ (04) is the condition for constructive inteference.

↑ Bragg's Law

Bragg Diffraction Example 1: Scattering from "120"-plane.



Find the lattice spacing D in terms of λ, n, φ.

From (14)

$2d\sin\theta = n\lambda$ (15)

$\varphi = 2\gamma$ (16)

$\theta + \gamma = 90^\circ \Rightarrow \theta = 90^\circ - \gamma$ (17)

(17) into (15) $\Rightarrow$ $2d\sin(90^\circ - \gamma) = n\lambda$

$2d\cos(\gamma) = n\lambda$ (18)

$\beta + \theta = 90^\circ$ (19)

$\Rightarrow$

(17) into (19) => $\beta + 90° - \gamma = 90°$

$$\Rightarrow \beta = \gamma : \quad (20)$$

$\sin\beta = \frac{d}{D}$ usin (20) => $\sin(\gamma) = \frac{d}{D} \Rightarrow d = D\sin(\gamma)$ (21)

(21) into (18) => $2D\sin(\gamma)\cos(\gamma) = n\lambda$

using $2\sin(x)\cos(x) = \sin(2x)$, yields

$$D\sin(2\gamma) = n\lambda$$

using (16) yields

$$\boxed{D\sin(\varphi) = n\lambda} \quad (22)$$

Numerical Calculation:

Given: $\varphi = 53.1°$, $\lambda = 0.165$ nm, $n=1$

Find D:

$$D = \frac{1 \cdot 0.165\,\text{nm}}{\sin(53.1°)} = \boxed{0.208\,\text{nm}}$$

DeBroglie's Matter-Wave Hypothesis

Einstein-Planck: Light has both wave and particle properties.

Remember for photons we have $E = \frac{hc}{\lambda}$

$$E = pc \Rightarrow p = \frac{E}{c} = \frac{h}{\lambda}$$

DeBroglie : "Nature should be symmetric"

i.e., Particles must have a complementary wave character (1924)

$$\boxed{\lambda = \frac{h}{p}} \quad (23)$$

Numerical Calculation: Wavelength of a bullet:

Gigen: $m = 1g$, $v = 300 \frac{m}{s}$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = 2.2 \cdot 10^{-33}\ m \quad \text{(very small)}$$

Numerical Calculation: Wavelength of e^- in H atom?

Given: $KE = 13.6\ eV$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mKE}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mKE}} = .34\ nm \quad (\sim \text{Å scale})$$

If electrons really do behave as waves, they should reveal wave phenomenon of <u>Interference</u>.

Lets work Bragg Diffraction Example 1 with e^- instead of light:

We found that $\varphi = 53.1°$, $\lambda = 0.165\,nm$; $n=1$

Find V_{acc}. Note: $KE = eV_{acc}$

(23) $\Rightarrow \lambda = \frac{h}{p}$; $p = \sqrt{2mKE} = \sqrt{2meV_{acc}}$

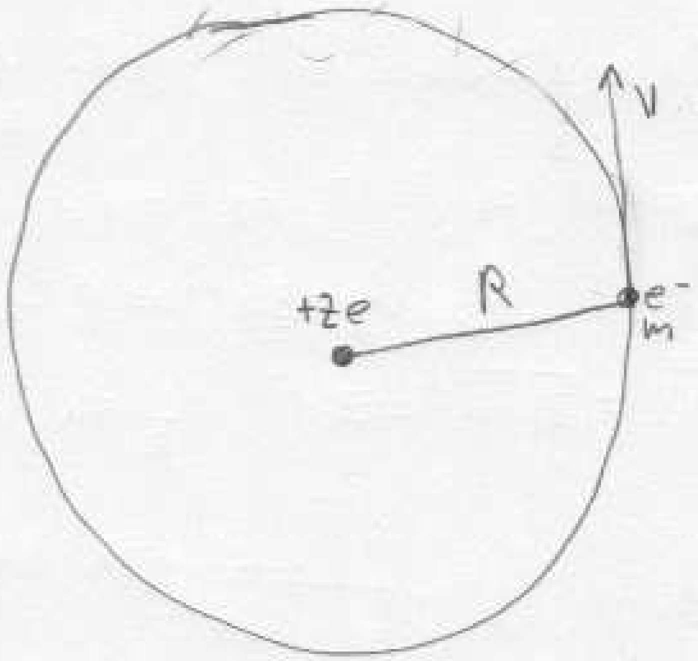
$$\Rightarrow \lambda = \frac{h}{\sqrt{2meV_{acc}}}$$

$$\Rightarrow V_{acc} = \frac{h^2}{2me\lambda^2} = \underline{\underline{55.3\,V}}$$

Davisson – Germer (1925).

Davisson and Thomson: Nobel Prize 1937.

Derivation of Bohr Hypothesis



Fit integer number of wavelengths into $(2\pi R)$.

$$\Rightarrow n\lambda = 2\pi R$$

From (23) we have $\lambda = \frac{h}{p}$

$$\Rightarrow n\frac{h}{p} = 2\pi R \quad ; \; p = mv$$

$$\Rightarrow n\frac{h}{2\pi} = Rmv \quad ; \; \vec{L} = \vec{r} \times \vec{p}$$

$$\Rightarrow L = Rmv$$

$$\Rightarrow L = n\frac{h}{2\pi}$$

Let: $\hbar = \frac{h}{2\pi}$, $\hbar$ is called h-bar.

$$\Rightarrow \boxed{L = n\hbar} \;\star$$