1. Those damn infinite and semi-infinite (half-infinite) wires!

   (a) Consider a semi-infinite wire carrying current I, see Fig. 1. Find the magnetic field, $\vec{B}$, in the first quadrant, i.e., $x \geq 0, y > 0, z = 0$. You may find

$$\int_0^\infty \frac{\eta}{\left[(\zeta - w)^2 + \eta^2\right]^{3/2}} \, \mathrm{d}w = \frac{1}{\eta} + \frac{\zeta}{\eta\sqrt{\zeta^2 + \eta^2}} \tag{1}$$

   useful.

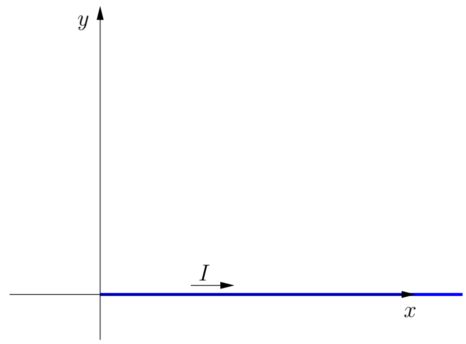

Figure 1: A semi-infinite wire (from 0 to ∞) carrying current I is shown in blue.

   Ans: $\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{1}{y} + \frac{x}{y\sqrt{x^2+y^2}}\right)\hat{\boldsymbol{k}}$.

   (b) Consider an infinite wire carrying current I, see Fig. 2. Find the magnetic field, $\vec{B}$, in the first quadrant, i.e., $x \geq 0, y > 0, z = 0$. You may find

$$\int_{-\infty}^\infty \frac{\eta}{\left[(\zeta - w)^2 + \eta^2\right]^{3/2}} \, \mathrm{d}w = \frac{2}{\eta} \tag{9}$$

   useful.

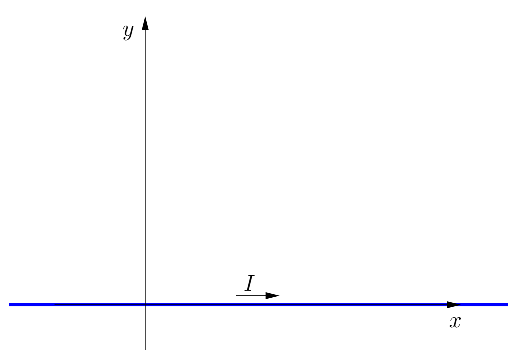

Figure 2: An infinite wire (from $-\infty$ to ∞) carrying current I is shown in blue.

   Ans: $\vec{B} = \frac{\mu_o I}{2\pi y}\hat{\boldsymbol{k}}$.

   (c) Where does the magnetic field due to a semi-infinite wire equal one half the magnetic field of an infinite wire, i.e., $\vec{B}_{\text{semi-infinite}} = \vec{B}_{\text{infinite}}/2$?

   Ans: $x = 0$, i.e, on the y-axis

2. A blue wire carrying current $I = I_o t^3/3$ is wound evenly on a torus of rectangular cross section. There are N turns of the blue wire in all. A red wire is thrown over the torus and is connected to a resistor, R, see Fig. 3. Find the magnitude and direction (clockwise or counterclockwise) of the current in the red wire, $I_{\text{red wire}}$.

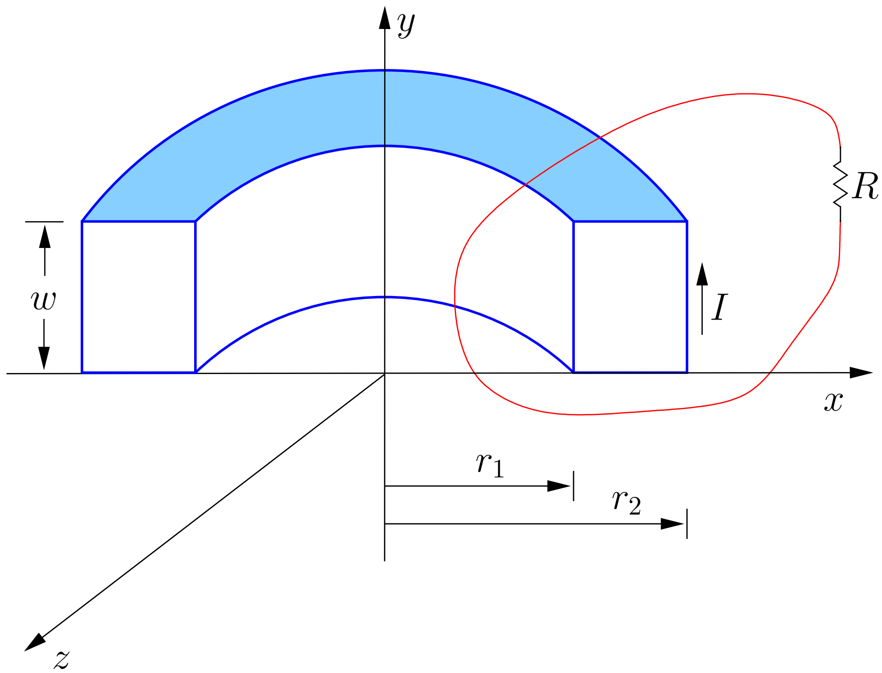

Figure 3: A blue wire carrying current $I = I_o t^3$ is wound evenly on a torus of rectangular cross section, with inner radius r_1 and outer radius r_2. There are N turns of the blue wire in all. A red wire is thrown over the torus and is connected to a resistor, R.

Ans: $I_{\text{red wire}} = \frac{\mu_o N w I_o}{2\pi R} \ln\left(\frac{r_2}{r_1}\right) t^2$, clockwise

3. Consider a conducting rod sitting on the top of an incline. The top of the incline is made from pair of frictionless conducting rails. There is a resistor, R, that connects the two rails, and a constant magnetic field directed vertically upwards with a magnitude B_o, see Fig. 5. The separation distance between the two frictionless conducting rails is L. If at time $t = 0$, the rod is released from rest, find the velocity of the rod as a function of time.

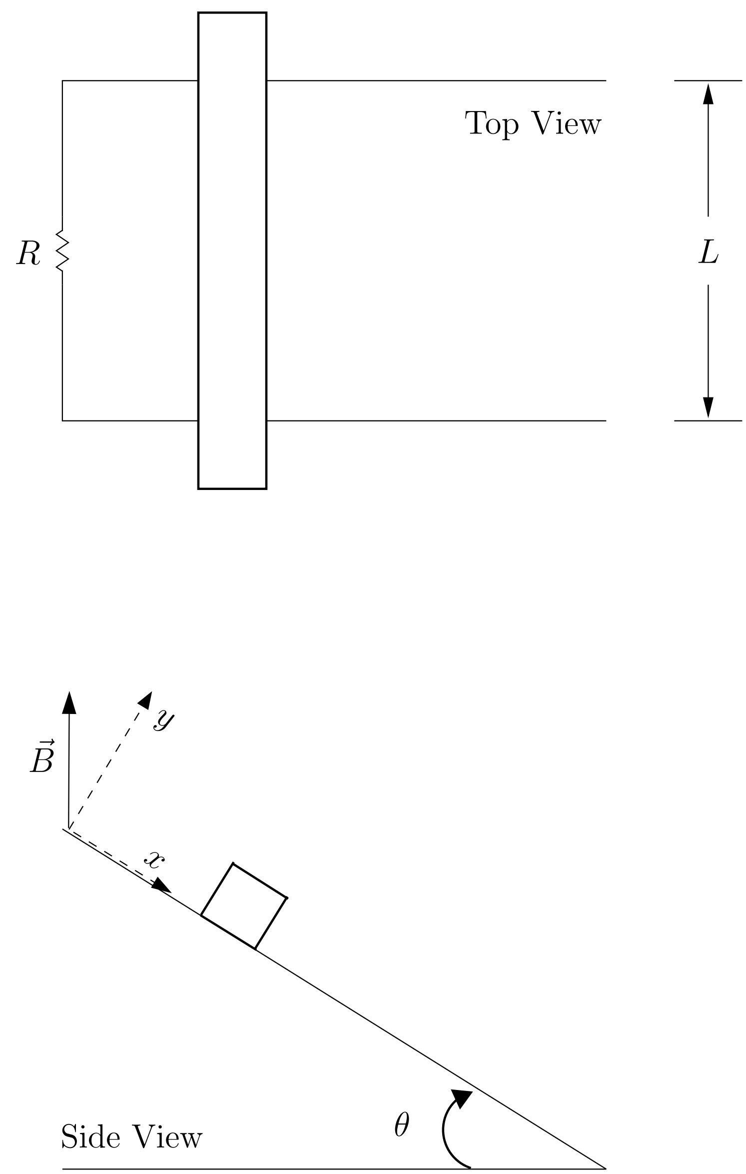

Figure 5: Top and side views of the conducting incline are shown. Notice that the pair of frictionless conducting rails, the conducting rod and the resistor form a complete circuit.

Ans: $v(t) = \frac{Rmg\sin\theta}{B_o^2 L^2 \cos^2\theta}\left(1 - e^{-\frac{B_o^2 L^2 \cos^2\theta}{Rm}t}\right).$