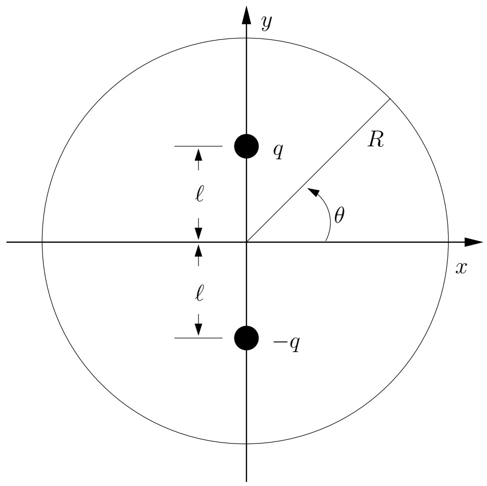

Figure 1: For the above electric dipole, $p = 2\ell q$.

1. Dipole in a 2-D world because the 3-D world is too damn hard!
   (a) Find the electric potential anywhere in the xy-plane (see Fig. 1).

   Ans: $V = \frac{q}{4\pi\epsilon_o}\left[\frac{1}{\sqrt{x^2+(y-\ell)^2}} - \frac{1}{\sqrt{x^2+(y+\ell)^2}}\right]$

   (b) Find the $x-$component of the electric field from the expression you have obtained for the electric potential in part (a). Is your result consistent with what you obtained during the **first** recitation?

   Ans: $E_x = \frac{q}{4\pi\epsilon_o}\left[\frac{x}{[x^2+(y-\ell)^2]^{3/2}} - \frac{x}{[x^2+(y+\ell)^2]^{3/2}}\right]$

2. Given a curve $y = f(x)$ where $x_1 \leq x \leq x_2$ in the $xy-$plane, find the electric potential at some field point, (a, b). Assume that the curve has a linear charge density given by $\lambda = g(x)$.

   Ans: $V = \frac{1}{4\pi\epsilon_o}\int_{x_1}^{x_2} \frac{\lambda}{\sqrt{(a-x)^2+(b-f(x))^2}}\sqrt{1+[f'(x)]^2}\,\mathrm{d}x, \quad \text{where}\lambda = g(x)$

3. A sphere with radius R and volume charge density $\rho = \rho_o r^n$, where $\rho_o > 0$ and $n \geq 0$, is centered at the origin of a coordinate system.
   (a) Find the electric field inside the sphere, i.e., $r < R$.

   Ans: $E = \frac{\rho_o r^{n+1}}{(n+3)\epsilon_o}, \quad r < R.$

   (b) Find the electric field outside the sphere, i.e., $r > R$.

   Ans: $E = \frac{\rho_o R^{n+3}}{r^2(n+3)\epsilon_o}, \quad r > R$

   (c) Find the electric potential difference between the center and surface of the sphere, i.e., $V_{\text{center}} - V_{\text{surface}}$.

   Ans: $V_{\text{center}} - V_{\text{surface}} = \frac{\rho_o R^{n+2}}{(n+2)(n+3)\epsilon_o}$

4. A cylinder with radius R, length ℓ, and volume charge density $\rho = \rho_o r^n$, where $\rho_o > 0$ and $n \geq 0$, is shown in Fig. (2).

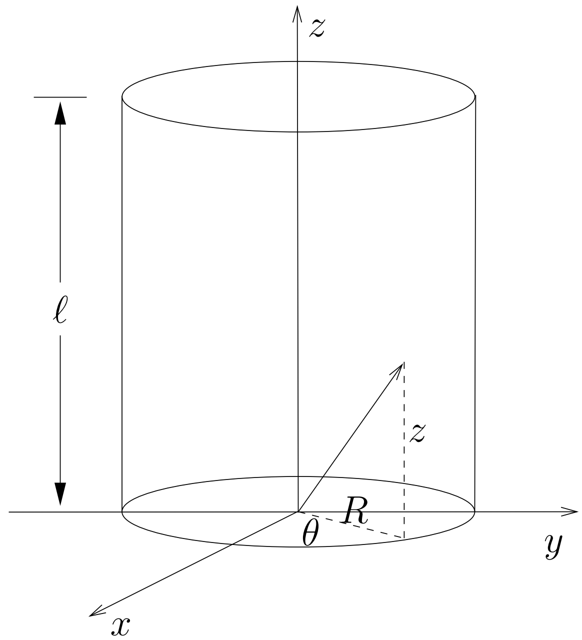

Figure 2: A cylinder with radius R, and length ℓ, where $\frac{R}{\ell} \ll 1$. The figure is not to scale.

(a) Find the electric field inside the cylinder, i.e., $r < R$.

Ans: $E = \frac{\rho_o r^{n+1}}{(n+2)\epsilon_o}, \quad r < R$

(b) Find the electric field outside the cylinder, i.e., $r > R$.

Ans: $E = \frac{\rho_o R^{n+2}}{(n+2)r\epsilon_o}, \quad r > R$

(c) Find the electric potential difference between the center and the surface of the cylinder, i.e., $V_{\text{center}} - V_{\text{surface}}$.

Ans: $V_{\text{center}} - V_{\text{surface}} = \frac{\rho_o R^{n+2}}{(n+2)^2\epsilon_o}$