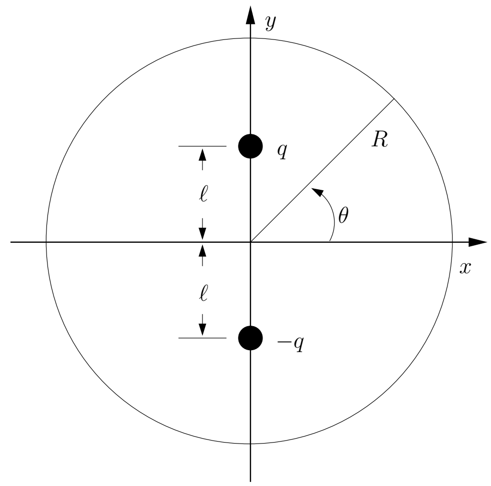

Figure 1: For the above electric dipole, $p = 2\ell q$.

1. Dipole in a 2-D world because the 3-D world is too damn hard!

   (a) Find the electric field anywhere in the xy-plane (see Fig. 1).

   Ans:
   $\vec{E} = \frac{q}{4\pi\epsilon_o}\left[\frac{x\hat{\imath}+(y-\ell)\hat{\jmath}}{[x^2+(y-\ell)^2]^{3/2}} - \frac{x\hat{\imath}+(y+\ell)\hat{\jmath}}{[x^2+(y+\ell)^2]^{3/2}}\right]$

   (b) Evaluate the electric field found in part (a) on a circle with radius R (see Fig. 1).

   Ans:
   $\vec{E} = \frac{q}{4\pi\epsilon_o}\left[\frac{R\cos\theta\hat{\imath}+(R\sin\theta-\ell)\hat{\jmath}}{[R^2-2\ell R\sin\theta+\ell^2]^{3/2}} - \frac{R\cos\theta\hat{\imath}+(R\sin\theta+\ell)\hat{\jmath}}{[R^2+2\ell R\sin\theta+\ell^2]^{3/2}}\right]$

   (c) Find an approximate expression for the **x-component** of the electric field on the circle if $\ell/R \ll 1$ (see Fig. 1). Hint: Expand the denominator in Taylor series, $(1+\epsilon)^n = 1 + n\epsilon + \cdots$, if $|\epsilon| < 1$. You can drop any terms containing square or higher powers of $\frac{\ell}{R}$ because if $\frac{\ell}{R}$ is small, then $\left(\frac{\ell}{R}\right)^2$ is super-small.

   Ans:
   $E_x = \frac{3p}{4\pi\epsilon_o R^3}\sin(\theta)\cos(\theta), \quad \text{where} \quad p = 2\ell q$

   (d) Find the total charge enclosed by the circle. Find the unit-normal to the circle.

   Ans: $q_{\text{enclosed}} = 0, \quad \hat{\boldsymbol{n}} = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$

2. Hard integrals made easy!

   (a) Set-up an integral expression for the electric field at a field point, $(x_o, 0)$, due to a ring of charge with linear charge density $\lambda = \lambda_o \sin\theta$, where λ_o is a **positive constant** (see Fig. 2).

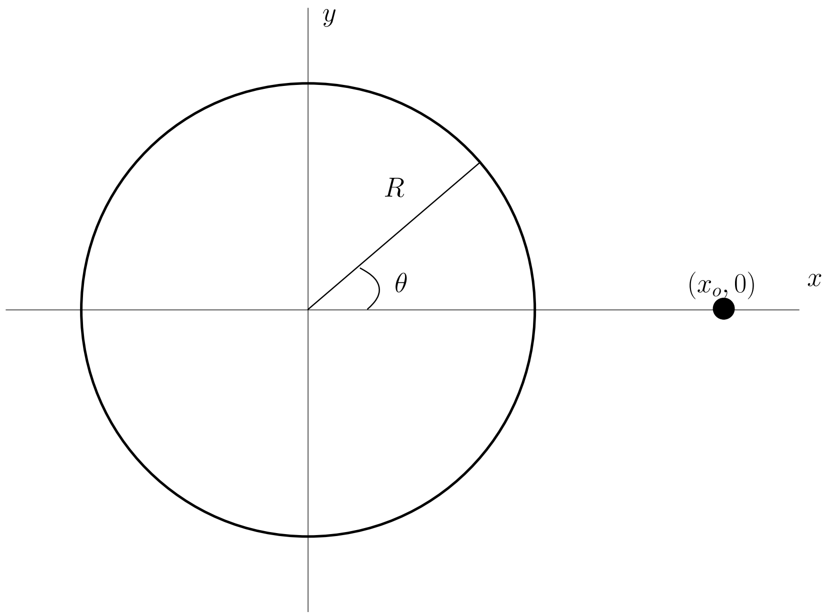

Figure 2: The circle has a linear charge density given by $\lambda = \lambda_o \sin\theta$, where λ_o is a **positive constant**.

Ans:
$\vec{E} = \frac{R}{4\pi\epsilon_o}\left[\int_0^{2\pi} \frac{\lambda(x_o - R\cos\theta)}{(R^2+x_o^2-2x_oR\cos\theta)^{3/2}}\mathrm{d}\theta\,\hat{\boldsymbol{\imath}} - \int_0^{2\pi} \frac{\lambda R\sin\theta}{(R^2+x_o^2-2x_oR\cos\theta)^{3/2}}\mathrm{d}\theta\,\hat{\boldsymbol{\jmath}}\right]$
where $\lambda = \lambda_o \sin\theta$

   (b) Using only a symmetry argument, find the direction of the electric field.
   Ans: in the negative y direction.

3. Given a curve $y = f(x)$ where $x_1 \le x \le x_2$ in the $xy-$plane, find the electric field at some field point, (a, b). Assume that the curve has a linear charge density given by $\lambda = g(x)$.
   Ans:
$\vec{E} = \frac{1}{4\pi\epsilon_o}\int_{x_1}^{x_2} \frac{\lambda(a-x)}{\left[(a-x)^2+(b-f(x))^2\right]^{3/2}}\sqrt{1+[f'(x)]^2}\,\mathrm{d}x\,\hat{\boldsymbol{\imath}} + \frac{1}{4\pi\epsilon_o}\int_{x_1}^{x_2} \frac{\lambda(b-f(x))}{\left[(a-x)^2+(b-f(x))^2\right]^{3/2}}\sqrt{1+[f'(x)]^2}\,\mathrm{d}x\,\hat{\boldsymbol{\jmath}}$