Name: ______________________________

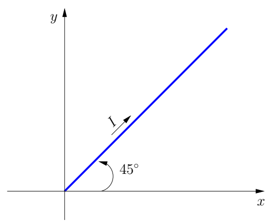
CWID: ______________________________

Calculators Not Allowed
No Work = No Credit
Write Legibly

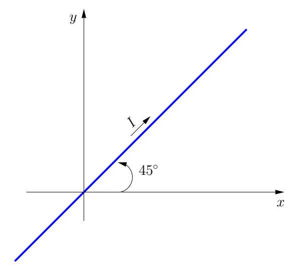
Question	Points	Score
1	6	
2	14	
Total:	20	

1. Quickies:

   (a) [3 points] Consider a semi-infinite wire carrying current I, see Fig. 1(a) and an infinite wire carrying current I, see Fig. 1(b). Where does the magnetic field due to a semi-infinite wire equal one half the magnetic field of an infinite wire, i.e., $\vec{B}_{\text{semi-infinite}} = \vec{B}_{\text{infinite}}/2$?



(a) A semi-infinite wire carrying current I is shown.



(b) An infinite wire carrying current I is shown.

Figure 1: A semi-infinite wire and an infinite wire are shown.

Solution:

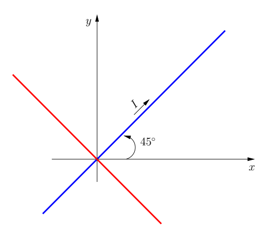


Figure 2: $\vec{B}_{\text{semi-infinite}} = \vec{B}_{\text{infinite}}/2$ on the red curve. Note: the red curve is perpendicular to the blue curve.

Table 1: For Grader Use Only: Rough grading criteria are given below.

red curve	3 pts

(b) 3 points Consider a conducting rod sitting on top of an incline. The top of the incline is made from a pair of frictionless conducting rails. There is a resistor, R, that connects the two rails, and a constant magnetic field directed vertically upward with a magnitude B_o, (same as the recitation problem). The separation distance between the two frictionless conducting rails is L. If at time $t = 0$, the rod is released from rest, explain physically why there is a flat region in Fig. 3.

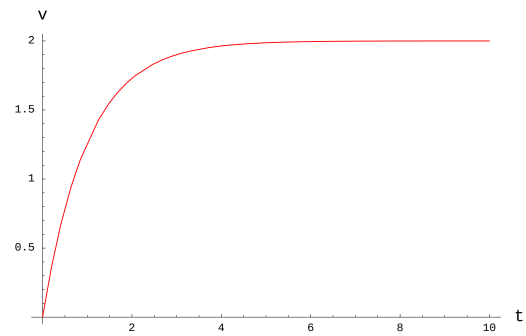


Figure 3: The velocity of the rod is shown for the following parameter values: $\frac{B_o^2 L^2 \cos^2\theta}{Rm} = 1$ and $g \sin\theta = 2$. Notice that the graph is flat for roughly $t > 5$.

Solution: The magnetic force is proportional to the speed of the rod. Thus, it's clear that the magnetic force will balance the gravity force after some time. This situation is analogous to a sky diver jumping out of an airplane, recall that air resistance is proportional to some power of the speed of the sky diver.

Table 2: For Grader Use Only: Rough grading criteria are given below.

$F_B \propto v$	3 pts

2. 14 points A blue wire carrying current $I = I(t)$ is wound evenly on a torus of rectangular cross section. There are N turns of the blue wire in all. A red wire is thrown over the torus and is connected to a resistor, R, see Fig. 4. The current in the red wire, $I_{\text{red wire}}$, is related to the current $I = I(t)$ via

$$|I_{\text{red wire}}| = \frac{M}{R}\left|\frac{\mathrm{d}I}{\mathrm{d}t}\right|. \tag{1}$$

Find the constant M in (1).

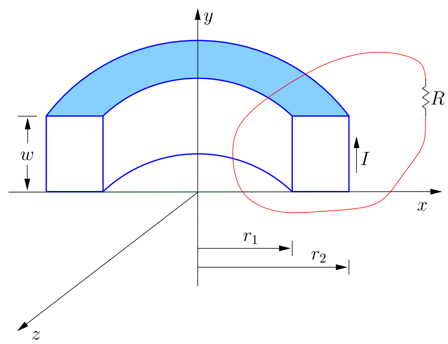


Figure 4: A blue wire carrying current $I = I(t)$ is wound evenly on a torus of rectangular cross section, with inner radius r_1 and outer radius r_2. There are N turns of the blue wire in all. A red wire is thrown over the torus and is connected to a resistor, R.

Solution: The magnetic field produced by the blue wire can be found via Ampere's Law,

$$\oint \vec{B}\cdot \mathrm{d}\vec{\ell} = \mu_o I_{\text{encl.}}, \tag{2}$$

where $I_{\text{encl.}}$ is the current enclosed by the Amperian loop. We choose the Amperian loop to be a circle of radius r centered at the origin (why?), see Fig. 5.

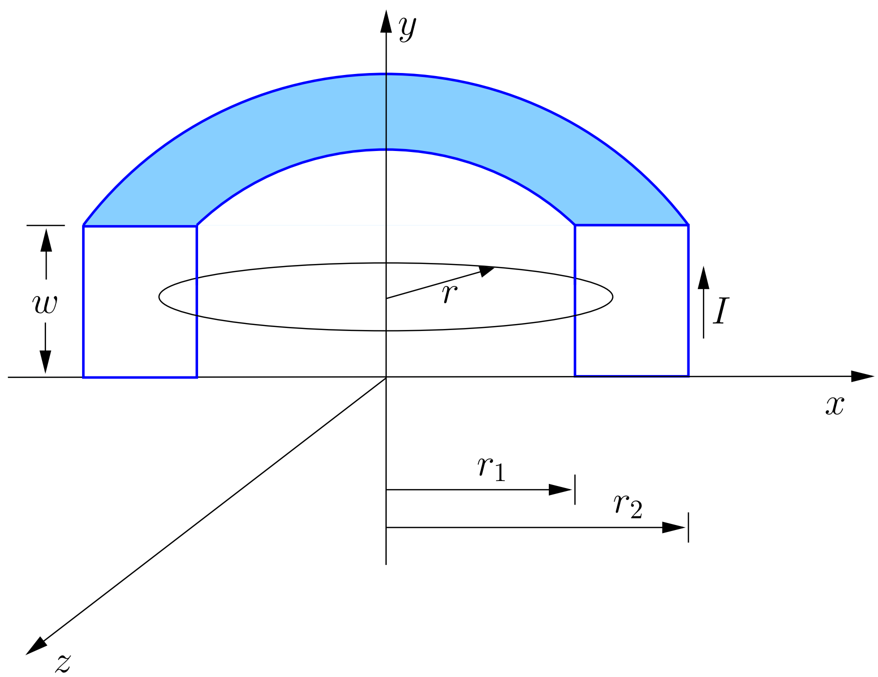


Figure 5: Amperian loop of radius r, laying "in the torus" is shown.

By symmetry, $\vec{B}$ is parallel to $\mathrm{d}\vec{\ell}$; thus, (2) yields

$$\oint B\mathrm{d}\ell = \mu_o I_{\text{encl.}}. \tag{3}$$

By symmetry, magnetic field B is constant on the Amperian loop; thus, (3) yields

$$\begin{aligned} B\oint \mathrm{d}\ell &= \mu_o I_{\text{encl.}} \\ B2\pi r &= \mu_o I_{\text{encl.}} \\ B &= \frac{\mu_o I_{\text{encl.}}}{2\pi r}, \end{aligned}$$

where $I_{\text{encl.}} = NI$. Thus, the magnetic field produced by the blue wire is given by

$$B = \frac{\mu_o NI}{2\pi r}. \tag{4}$$

The magnetic flux through the **area enclosed by the red wire** is given by

$$\begin{aligned} \Phi_B &= \int \vec{B}\cdot \mathrm{d}\vec{A} \\ &= \int_{r_1}^{r_2} B\, w\mathrm{d}r. \end{aligned} \tag{5}$$

Why are the limits of integration from r_1 to r_2 if we are calculating the magnetic flux through the **area enclosed by the red wire**? Substituting (4) into (5) and integrating yields

$$\Phi_B = \frac{\mu_o N I w}{2\pi} \ln\left(\frac{r_2}{r_1}\right). \tag{6}$$

The magnitude of the induced emf is given by

$$|\mathcal{E}| = \left|\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}\right|. \tag{7}$$

Substituting (6) into (7) yields

$$|\mathcal{E}| = \frac{\mu_o N w}{2\pi} \ln\left(\frac{r_2}{r_1}\right) \left|\frac{\mathrm{d}I}{\mathrm{d}t}\right| \tag{8}$$

Substituting (8) into Ohm's law yields

$$|I_{\text{red wire}}| = \frac{\mu_o N w}{2\pi R} \ln\left(\frac{r_2}{r_1}\right) \left|\frac{\mathrm{d}I}{\mathrm{d}t}\right|,$$

where $I_{\text{red wire}}$ flows in the clockwise direction (why?). Thus,

$$M = \frac{\mu_o N w}{2\pi} \ln\left(\frac{r_2}{r_1}\right) \tag{9}$$

Table 3: For Grader Use Only: Rough grading criteria are given below.

Eqn. (4)	5 pts
Eqn. (6)	5 pts
Eqn. (9)	4 pts