Name: ______________________________

CWID: ______________________________

Calculators Not Allowed
No Work = No Credit
Write Legibly

Question	Points	Score
1	10	
2	10	
Total:	20	

1. [10 points] A particle with charge q is traveling with constant velocity $\vec{v}$, where $v \ll c$ and $c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$ is the speed of light (we will learn this later in the course). Write an expression for the magnetic field, $\vec{B}$, that the particle generates, in terms of the electric field $\vec{E}$ and speed of light c.

Solution: The magnetic field due to a slow moving charge is **approximately** (here, "approximately" means we are ignoring retardation effects; don't worry if you don't know what this means) given by

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}, \tag{1}$$

and the electric field is given by

$$\vec{E} = \frac{q}{4\pi\epsilon_o} \frac{\vec{r}}{r^3}. \tag{2}$$

Solving (2) for $\frac{\vec{r}}{r^3}$ and substituting the result into (1) yields

$$\vec{B} = \frac{\vec{v} \times \vec{E}}{c^2}. \tag{3}$$

Equation (3) is a "cute" equation relating the electric and magnetic fields. Later on in the course, we will learn that electric and magnetic fields are fundamentally related quantities; this is largely due to the work of James Clerk Maxwell.

Table 1: For Grader Use Only: Rough grading criteria are given below.

Eqn. (1)	3 pts
Eqn. (2)	3 pts
$c = 1/\sqrt{\epsilon_o \mu_o}$	1 pt
Eqn. (3)	3 pts

2. [10 points] Set up an expression for the net force on the wire (curve) $y = f(x)$, where $x_1 \leq x \leq x_2$ lies in the $z = 0$ plane and carries a current I (see Fig. 1). Assume that there is a non-uniform magnetic field $\vec{B} = \vec{B}(x, y, z)$ in the region of space where the wire lies.

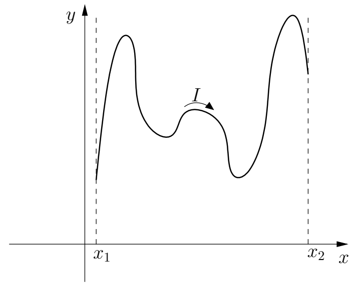

Figure 1: An arbitrary wire (curve), $y = f(x)$, carrying a current I (as indicated on the diagram) is shown.

Solution:

The magnetic force on the wire is given by

$$\mathrm{d}\vec{F} = I\,\mathrm{d}\vec{\ell} \times \vec{B}, \tag{4}$$

where $\mathrm{d}\vec{\ell}$ is computed in the "usual way", i.e.,

$$\begin{aligned}
\vec{\ell} &= x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} \\
\vec{\ell} &= x\hat{\boldsymbol{\imath}} + f(x)\hat{\boldsymbol{\jmath}} \\
\frac{\mathrm{d}\vec{\ell}}{\mathrm{d}x} &= 1\hat{\boldsymbol{\imath}} + f'(x)\hat{\boldsymbol{\jmath}} \\
\mathrm{d}\vec{\ell} &= \mathrm{d}x\hat{\boldsymbol{\imath}} + f'(x)\,\mathrm{d}x\hat{\boldsymbol{\jmath}},
\end{aligned} \tag{5}$$

where $f'(x)$ denotes a derivative of $f(x)$ w.r.t x. Evaluating $\vec{B}$ on the wire (curve) yields

$$\begin{aligned}
\vec{B} &= \vec{B}(x, y, z) \\
\vec{B} &= \vec{B}(x, y = f(x), z = 0).
\end{aligned} \tag{6}$$

Finally, substituting (5) and (6) into (4) yields

$$\begin{aligned}
\mathrm{d}\vec{F} &= I\big[\mathrm{d}x\hat{\boldsymbol{\imath}} + f'(x)\,\mathrm{d}x\hat{\boldsymbol{\jmath}}\big] \times \vec{B}(x, y = f(x), z = 0) \\
\vec{F} &= \int_{x_1}^{x_2} I\big[\mathrm{d}x\hat{\boldsymbol{\imath}} + f'(x)\,\mathrm{d}x\hat{\boldsymbol{\jmath}}\big] \times \vec{B}(x, y = f(x), z = 0).
\end{aligned} \tag{7}$$

Table 2: For Grader Use Only: Rough grading criteria are given below.

Eqn. (5)	4 pts
Eqn. (6)	4 pts
Eqn. (7)	2 pt