Name: ______________________

CWID: ______________________

Question	Points	Score
1	10	
2	10	
Total:	20	

Calculators Not Allowed
No Work = No Credit
Write Legibly

1. [10 points] An experimentalist observes a particle of charge q, mass m, and energy qV_o moving in a circular orbit of radius r_o inside a **spherical** capacitor with inner radius a and outer radius b, see Fig. (1).

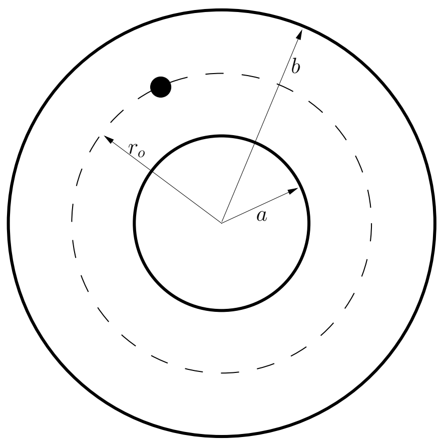

Figure 1: A particle of charge q, mass m, and energy qV_o moving inside a **spherical** capacitor is shown.

The experimentalist quickly observes that the particle only moves in a circular orbit with radius r_o when $V_a - V_b$ has a very special value. Find this value. **Start with Gauss's Law.**

Solution: Let Q be the amount of positive charge on the inner sphere; the electric field between the two spheres is given by (use Gauss's Law to show this or look at the Recitation 2 problems)

$$E = \frac{Q}{4\pi\epsilon_o r^2}, \quad a < r < b, \tag{1}$$

The electric potential difference, $V_a - V_b$, is given by (see Recitation 2 problems)

$$\begin{aligned} V_a - V_b &= \int_a^b \vec{E}\cdot \mathrm{d}\vec{\ell} \\ &= \int_a^b \vec{E}\cdot \mathrm{d}x\boldsymbol{\hat{\imath}} \\ &= \int_a^b E\mathrm{d}x \\ &= \frac{Q}{4\pi\epsilon_o}\int_a^b \frac{1}{x^2}\,\mathrm{d}x \\ &= \frac{Q}{4\pi\epsilon_o}\left(\frac{b-a}{ab}\right). \end{aligned}$$

Thus,

$$Q = \frac{4\pi\epsilon_o ab}{(b-a)}\left(V_a - V_b\right). \tag{2}$$

Setting the particle's kinetic energy equal to qV_o yields

$$v^2 = \frac{2qV_o}{m}, \tag{3}$$

where v is the particle's speed. Finally, using (1), (2), (3), and Newton's Second Law yields

$$\begin{aligned} qE &= m\frac{v^2}{r_o} \\ q\left[\frac{4\pi\epsilon_o ab}{(b-a)}\left(V_a - V_b\right)\right]\left[\frac{1}{4\pi\epsilon_o r_o^2}\right] &= \frac{m}{r_o}\left[\frac{2qV_o}{m}\right] \\ V_a - V_b &= \frac{2V_o r_o\left(b-a\right)}{ab} \end{aligned}$$

Table 1: For Grader Use Only: Rough grading criteria are given below.

Computation of E field	3 pts
Computation of $V_a - V_b$	3 pts
Energy of the particle, $\frac{mv^2}{2} = qV_o$	2 pts
Newton's Law and algebra	2 pts

2.

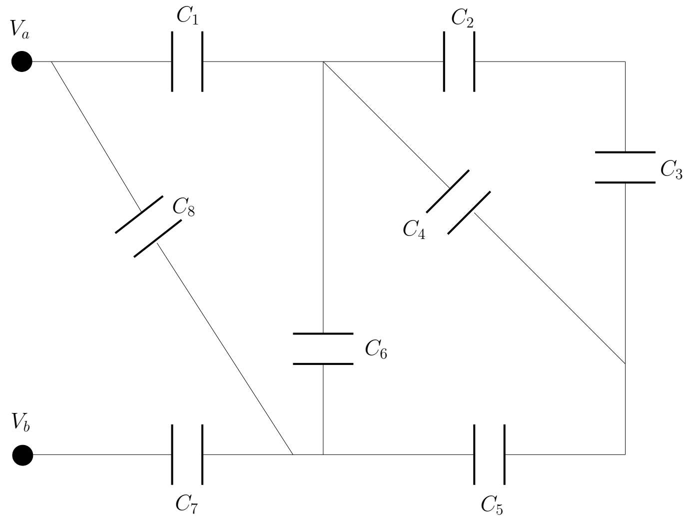

Figure 2: A capacitor network is shown.

(a) 7 points Find the equivalent capacitance, C_{eq}, of the circuit shown in Fig. (2). You may use $\perp$, $\|$ symbolism.

Solution:

$$C_9 = C_2 \perp C_3 \tag{4}$$
$$C_{10} = C_9 \| C_4 \tag{5}$$
$$C_{11} = C_5 \perp C_{10} \tag{6}$$
$$C_{12} = C_6 \| C_{11} \tag{7}$$
$$C_{13} = C_1 \perp C_{12} \tag{8}$$
$$C_{14} = C_8 \| C_{13} \tag{9}$$
$$C_{\text{eq}} = C_{14} \perp C_7 \tag{10}$$

Table 2: For Grader Use Only: Rough grading criteria are given below.

Each numbered equation	1 pt

(b) 3 points Give formulas for computing $a \perp b$ and $a \| b$.

Solution:

$$a \perp b = \left(a^{-1} + b^{-1}\right)^{-1} \tag{11}$$
$$a \| b = a + b \tag{12}$$

Table 3: For Grader Use Only: Rough grading criteria are given below.

Each numbered equation	1.5 pts