Solutions

No Work = No Credit. Write Legibly. Box your final result.

1. $\boxed{\text{5 points}}$

$$f(x) = x^4 - 2x^2 + 3$$

(a) $\boxed{\text{2 points}}$ Find the intervals on which f is increasing or decreasing.
To find critical numbers, we set the derivative of $f(x)$ to equal zero, to obtain

$$4x\left(x^2 - 1\right) = 0. \tag{1}$$

Solving equation (1) for x, we obtain the critical values of $f(x)$, which are given by $x = 0, \pm 1$. $\boxed{f(x) \text{ is decreasing on } (-\infty, -1)}$, because $\lim_{x \to -\infty} f'(x) = -\infty$. $\boxed{f(x) \text{ is increasing on } (-1, 0)}$, because $f'(-1/2) = 3/2$. $\boxed{f(x) \text{ is decreasing on } (0, 1)}$, because $f'(1/2) = -3/2$. And finally we have that, $\boxed{f(x) \text{ is increasing on } (1, \infty)}$, because $\lim_{x \to \infty} f'(x) = \infty$.

(b) $\boxed{\text{1 point}}$ Find the local maximum and minimum values of f.
Local mins. happen when f switches from decreasing to increasing, thus local mins. are given by

$$\boxed{f(\pm 1) = 2}.$$

Local maxs. happen when f switches from increasing to decreasing, thus local maxs. are given by

$$\boxed{f(0) = 3}.$$

(c) $\boxed{\text{2 points}}$ Find the intervals of concavity and the inflection points.
To find *possible* inflection points of $f(x)$, we set the second derivative of $f(x)$ to equal zero, to obtain

$$12x^2 - 4 = 0. \tag{2}$$

Solving equation (2) for x, we obtain possible inflection points of $f(x)$, which are given be $x = \pm 1/\sqrt{3}$. Using our answers from part (a) of the question, we conclude that $\boxed{\text{f(x) in concave up on } (-\infty, -1/\sqrt{3}),\ (1/\sqrt{3}, \infty)}$, and that $\boxed{f(x) \text{ is concave down on } (-1/\sqrt{3}, 1/\sqrt{3})}$. We now see that $f(x)$ changes concavity at $\pm 1/\sqrt{3}$, therefore, we conclude that $\boxed{x = \pm 1/\sqrt{3} \text{ are indeed inflection points of } f(x)}$.

2. 5 points Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$\lim_{x\to 0^+} x^{x^2} \tag{3}$$

First we note that (3) is *not* in l'Hospital Rule form. To obtain l'Hospital Rule form from (3) we do the following. Let

$$y = x^{x^2}, \tag{4}$$

then taking natural log of both sides of (4), yields

$$\ln(y) = x^2 \ln(x). \tag{5}$$

Notice that (5) is *not* in l'Hospital form yet. To obtain l'Hospital form of (5), we write x^2 as $1/x^{-2}$ to obtain

$$\ln(y) = \frac{\ln(x)}{x^{-2}}. \tag{6}$$

Finally we see that (6) is in l'Hospital form. Taking $\lim_{x\to 0^+}$ of both sides of (6) and using l'Hospital Rule, yields

$$\begin{aligned}\lim_{x\to 0^+} \ln(y) &= \lim_{x\to 0^+} \frac{x^{-1}}{-2x^{-3}} \\ &= \lim_{x\to 0^+} -\frac{1}{2}x^2 \\ &= 0. \end{aligned} \tag{7}$$

Finally, solving (7) for y, yields $\lim_{x\to 0^+} y = e^0 = 1$, i.e.,

$$\boxed{\lim_{x\to 0^+} x^{x^2} = 1}.$$