Solutions

No Work = No Credit. Write Legibly. Box your final result.

1. 10 points Prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\csc^2 x. \tag{1}$$

Rewriting (1) in terms of sines and cosines for simplicity, yields

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{\sin x}\right) = -\frac{1}{\sin^2 x}. \tag{2}$$

Thus, proving (1) is the same as proving (2) because they are the same equation just written differently. We will prove (2) because it is simpler.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{\sin x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left[\cos x(\sin x)^{-1}\right] \\
&= -\sin x\,(\sin x)^{-1} + \cos x\left[-(\sin x)^{-2}\cos x\right] \\
&= -1 - \frac{\cos^2 x}{\sin^2 x} \\
&= -\left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right) && (3) \\
&= \boxed{-\frac{1}{\sin^2 x}}. && (4)
\end{aligned}$$

Notice that we used $\sin^2 x + \cos^2 x = 1$ identity to obtain (4) from (3). Finally, by comparing (4) with (2), we conclude that (1) must be true.