Solutions

No Work = No Credit. Write Legibly. Box your final result.

1. [10 points] If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Since the base of the box is a square, we let x be the length of any side of the square. Thus, the area of the base is given by x^2. Let y be the height of the box (do *not* assume that the box is a cube). The box is open (no top) thus, total surface area of the box (4 sides and the base) is given by

$$A = x^2 + 4xy.$$

From the problem statement, we know that $A = 1200$ cm^2 thus, we have the following constraint relationship

$$1200 = x^2 + 4xy. \tag{1}$$

The volume of the box is given by

$$V = x^2 y. \tag{2}$$

Solving (1) for y and substituting it into (2) yields

$$V = x^2 \left(\frac{1200 - x^2}{4x} \right). \tag{3}$$

To find the maximum volume, we take a derivative of (3) with respect to x and set the result equal to zero, to obtain

$$300 - \frac{3x^2}{4} = 0. \tag{4}$$

Solving (4) for x, yields $x = 20$ cm. To find the y value, we substitute $x = 20$ cm into (1), to obtain $y = 10$ cm. Finally, substituting $x = 20$ cm and $y = 10$ cm into (2) yields

$$\boxed{V = 4000 \text{ cm}^3}.$$